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Compensating arithmetic ability with derived fact strategies in Broca's aphasia: a case report

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ABSTRACT
We investigated derived fact strategy use in RR, an aphasic patient with severely impaired working memory (no phonological loop), and 16 neurologically healthy matched controls. Participants were tested on derived fact strategy use in multi-digit addition, subtraction, multiplication, and division. RR's accuracy only differed from controls in multiplication. He was as quick as controls in addition and subtraction when able to use the strategies, though significantly slower in addition, division, and multiplication without strategies. Our findings suggest the phonological loop is non-essential for multi-digit arithmetic, and derived fact strategies can help speed up arithmetic in individuals with impaired working memory.

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Broca's aphasia; verbal working memory; arithmetic; derived fact strategies; stroke

Neuropsychological findings support distinctions between at least three types of arithmetical knowledge: procedural, factual, and conceptual (Delazer, 2003). Procedural knowledge involves knowing how to use certain arithmetic strategies, factual knowledge involves knowing certain arithmetical facts by rote, and conceptual knowledge involves understanding the mathematical principles underlying operations and strategies. Patients with selective impairments in arithmetic have been known to use a wide range of compensatory strategies, varying in complexity, to solve arithmetic problems (Delazer & Benke, 1997; Hittmair-Delazer, Sailer, & Benke, 1995; Hittmair-Delazer, Semenza, & Denes, 1994; McCloskey, Aliminosa, & Sokol, 1991; Warrington, 1982).

Brain-damaged patients with selective mathematical impairments have provided evidence for a double dissociation between arithmetical fact retrieval and the understanding and use of arithmetic strategies (Delazer & Benke, 1997; Hittmair-Delazer et al., 1994). Patient MW (McCloskey, Caramazza, & Basili, 1985) showed impaired fact retrieval, particularly for multiplication facts, but was able to use various strategies to correctly carry out calculations. Hittmair-Delazer et al. (1994) described an aphasic patient, BE, who used complex strategies to compensate for his deficit in multiplication fact retrieval. Similar dissociations have been found in patients with degenerative disorders such as semantic dementia (Cappelletti, Butterworth, & Kopelman, 2012) and Alzheimer's disease (Duverne, Lemaire, & Michel, 2003), where patients presented with impaired fact retrieval yet relatively intact understanding and implementation of arithmetical strategies.

Strategy use in mental arithmetic is strongly influenced by, and to some degree dependent on, working memory (see DeStefano & LeFevre, 2004; Raghubar, Barnes, & Hecht, 2010 for reviews). Working memory refers to the resource used to keep things actively in mind and organize mental processes during tasks. Baddeley and Hitch (1974) proposed a highly influential multi-component model of working memory, consisting of the central executive, phonological loop, and visuo-spatial sketchpad. The model was updated (Baddeley, 2000) to include the episodic buffer, which stores temporary, multidimensional representations and can integrate information from long-term memory and the two short-term stores. Logie (1995, 2011) suggested that visuo-spatial working memory can be further subdivided into the visual cache (visual short-term memory) and an inner scribe which allows the individual to keep in mind short sequences of movements through mental rehearsal. He described the corresponding phonological mechanism for subvocal rehearsal as inner speech.

The central executive, phonological loop, and visuo-spatial sketchpad appear to be differentially related to particular mathematical skills (Simmons, Willis, & Adams, 2012). Wilson and Swanson (2001) found that both domain-general and domain-specific working memory deficits were linked to mathematical deficits. In multi-digit arithmetic, which tends to require carrying (incrementing) or borrowing (decrementing) across numerical columns, the central executive is most important (Fürst & Hitch, 2000; Hubber, Gilmore, & Cragg, 2014; Imbo, Vandierendonck, & De Rammelaere, 2007; Logie, Gilhooly, & Wynn, 1994). Studies suggest that the phonological loop also plays a key role in multi-digit arithmetic for maintaining intermediate results (Heathcote, 1994; Noël, Désert, Aubrun, & Seron, 2001; Seitz & Schumann-Hengsteler, 2000), and maintaining operands that are presented briefly (Fürst & Hitch, 2000). The phonological loop may be more involved in multiplication than simple subtraction, which seems to rely more on the visuo-spatial sketchpad (Lee & Kang, 2002). Seyler, Kirk, and Ashcraft (2003), however, found that the phonological loop was implicated in solving larger, multi-digit subtraction problems.

Working memory is important in the selection, execution, and adaptiveness of strategies in arithmetic (Imbo, Duverne, &

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Lemaire, 2007). Imbo and Vandierendonck (2007) found that the central executive and active phonological processes were involved in carrying out procedural strategies, including transformation and counting strategies.

The present study aimed to compare the use of derived fact strategies in a single-case aphasic patient who lacked a phonological loop, RR, with that of matched controls. RR, whose neurological and cognitive deficits are described in the Method section, was a patient with Broca's aphasia, an apparent lack of phonological working memory, and a very low digit span even when tested visually. Although they were not given exactly the same tests, his phonological deficits resemble those of patient PV (Vallar & Baddeley, 1984, 1987).

Derived fact strategies involve using one or more known number facts, combined with some knowledge about relationships between numbers or operations, to derive the solution to unknown number facts. Although researchers have investigated the use of derived fact strategies in children (e.g., Baroody, Ginsburg, & Waxman, 1983; Dowker, 1998, 2009, 2014), and there have been studies showing forms of derived fact strategy use in brain-damaged patients (e.g., Hittmair-Delazer et al., 1994; Warrington, 1982), there is to our knowledge no published work on derived fact strategy use in patients with severely impaired working memory associated with expressive aphasia.

The derived fact strategies investigated in this study were based on six principles across the four mathematical operations. "Principles" are referred to here as shorthand for knowledge about relationships between numbers and operations that can facilitate arithmetic; this study is not concerned with the details of the cognitive architecture that supports such knowledge. People are assumed to be using the principles at some level if they consistently perform better on tasks where they are presented with an initial fact from which they could derive the new fact with the aid of the principle, than when they are not presented with such an initial fact. No assumptions are made about the manner or conceptual depth with which they use the principle, for example, whether this involves abstract reasoning, rule use, or analogy use.

The first hypothesis was that both RR and healthy controls would answer mental arithmetic questions faster when there was an opportunity to use a derived fact strategy (based on one of the six principles) than when there was no such opportunity. As RR had impaired working memory, fact retrieval, and language, it was predicted that he would be more reliant on compensatory strategies compared to the healthy population. Therefore, the second hypothesis was that RR would show a significantly larger difference in response times with versus without the use of derived fact strategies compared to healthy controls.

Case report

RR was 32 years old at the time of testing. He suffered a left middle cerebral artery ischemic stroke at the age of 31, resulting in Broca's aphasia. An MRI scan taken one year post-stroke showed large lesions to his left temporal lobe (see Figure 1), including the inferior, middle, and superior temporal gyrus, angular gyrus, temporal pole, and extending into insular cortex, supramarginal gyrus, frontal and central operculum cortex, inferior frontal gyrus, as well as parietal operculum. RR was initially seen two weeks post-stroke, with complete expressive aphasia and some problems comprehending complex instructions.

Eight months later, his performance on the Oxford Cognitive Screen (Demeyere, Riddoch, Slavkova, Bickerton, & Humphreys, 2015) showed that he was still impaired on reading, naming, writing numbers, and verbal memory (see Figure 2(a)). We also report RR's performance on the Birmingham Cognitive Screen (BCoS) (Humphreys, Bickerton, Samson, & Riddoch, 2012), covering five domains of cognition: Memory, Number, Language, Praxis, and Spatial and Controlled Attention, in a visual snapshot in Figure 2(b). RR was administered the Western Aphasia Battery 11 months post-stroke. He showed severe deficits on all four expression tasks, and made errors in understanding function words in the Auditory Comprehension tasks. He performed well in the Written Comprehension tasks (see Table 1 for values).

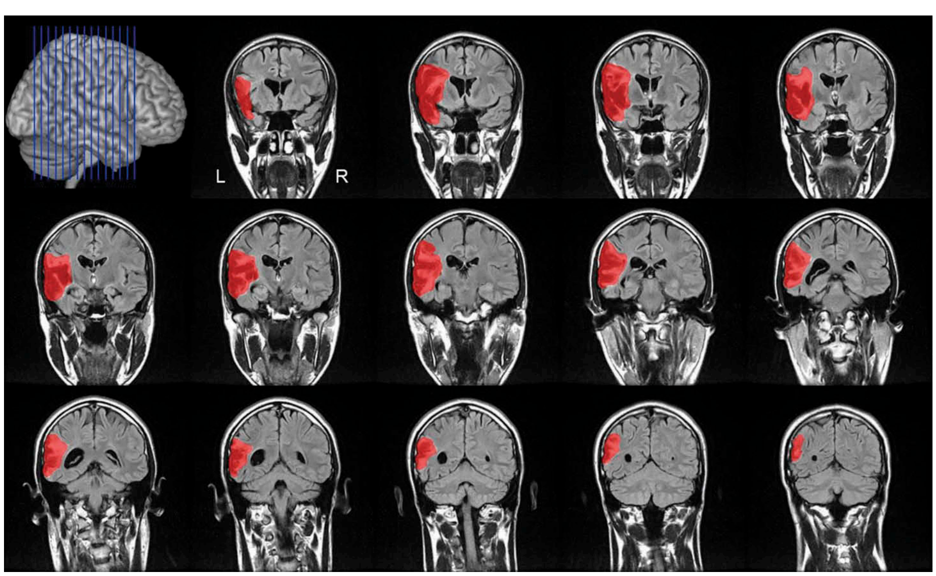

Figure 1. MRI scan: T2 Flair image with an enhanced manual overlay of the lesion.

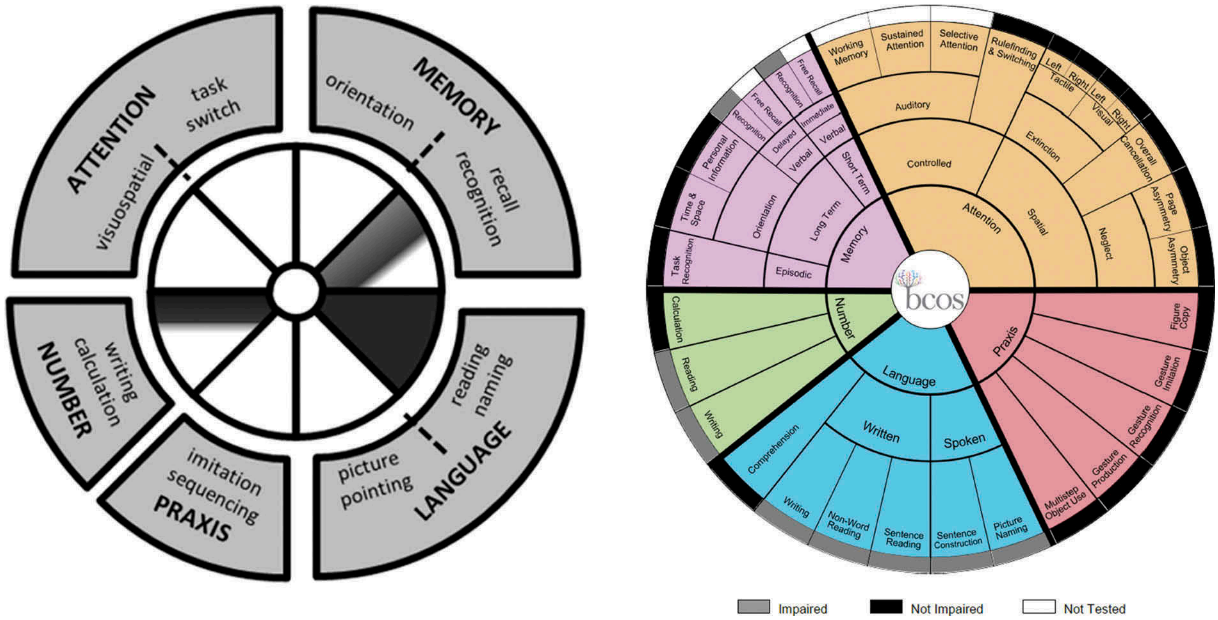

Figure 2. (a) Oxford cognitive screen showing RR's performance eight months post-stroke. Shaded areas of the inner circle indicate impairment. (b) BCoS cognitive profile of five cognitive domains, with grey areas of the outer circle denoting impairments. The tasks that were not tested were all due to RR's expressive aphasia, in particular: free recall and working memory using a phonological loop in the auditory attention task.

Table 1. RR's performance on the Western Aphasia Battery, 11 months post-stroke.

Task	Score
Expression	
Spontaneous speech	6/20
Repetition	8/100
Fluency	8/20
Reading aloud	7/20
Auditory Comprehension	
Auditory Verbal comprehension	54/60
Sequential commands	75/80
Written Comprehension	
Reading of sentences	40/40
Word/word matching	6/6
Word/picture matching	6/6

His score on Raven's progressive matrices (Raven, 1976) fell in the 95th centile, demonstrating a very high non-verbal IQ.

As noted from the BCoS, RR was impaired in number reading and in writing numbers from auditory input (dictation by examiner). In contrast, his basic numerical skills were good when not requiring verbal output or auditory input. For non-symbolic numerosities, RR was able to correctly count the number of random dots on a screen (30 trials per numerosity, accuracy 100%), and estimate quantities (which has more?) using the Panamath task (Halberda, Mazzocco, & Feigenson, 2008) at levels within the normal range for his age. For symbolic numbers, RR was 100% correct when transcoding between numerical (Arabic) formats, dot quantities, and written formats of numbers under 20 (20 trials per numerosity, all performance at ceiling). When doing simple sums (both Arabic notation and written numbers under 20), his accuracy was at ceiling on all tests (responses via keyboard, not spoken). RR was able to correctly complete multiplication and division tasks, though he was quite slow and explained he used various strategies to do this, so verbal fact retrieval did not appear intact despite very high accuracy (59/64 trials correct overall with eight trials per times table, ranging from table of two to table of nine).

This study took place 18 months after RR's stroke. There was no evidence of RR having a phonological loop. He was given a visual input digit span test in which he was shown a sequence of numbers and had to point to the same sequence of numbers using a multiple choice format. RR had a very low visual input digit span of two. His digit span with auditory input was only one. He reported that he had no "internal speech". RR was unable to verbalize his responses to questions. His premorbid knowledge of mathematics was very good; he holds a degree in law and accounting and worked as a business analyst prior to his stroke.

Method

Participants

In addition to RR, 16 healthy control participants (nine men and seven women) aged between 28 and 35 years participated in this study. The control participants were matched in age to RR (aged 32) and their mean age was 31 years and 8 months (SD = 2 years and 1 month). Participants were also matched in years of education, with all participants having higher university degrees. All participants were right handed and had self-reported normal or corrected-to-normal vision.

Apparatus

The tests in this experiment were programmed and run using E-Prime 2.0 software (Schneider, Eschman, & Zuccolotto, 2002). Questions were horizontally displayed in white on a black background and a computer keyboard with a numeric keypad was used to input responses. Response times were recorded once the participant hit the spacebar, before entering their response.

Table 2. Overview of the six principles investigated.

Principle	Description
$N + 3$	If one addend increases by three, the sum of both addends will also increase by three
$N + 10$	If one addend increases by 10, the sum will also increase by 10
$N - 1$	If one of the addends decreases by one, the sum will also decrease by one
Addition/Subtraction Inverse (ASI)	If $a + b = c$, then $c - b = a$
Multiplication/Division Inverse (MDI)	If $a \times b = c$, then $c \div b = a$
$M + 1$	If one of the operands increases by one, the sum increases by the other operand. For half of the trials the multiplicand increased by one, and for the other half the multiplier increased by one.

Procedure

The tests used in this experiment were similar to those used by Dowker (1998, 2009, 2014), who investigated derived fact strategy use (for addition and subtraction) in children, though a wider variety of principles were studied in this experiment. An overview of the principles is presented in Table 2. Three associativity-based principles for addition were investigated: the $N + 3$ principle (e.g., if 21 + 53 = 74, 24 + 53 must be 74 + 3 or 77), the $N + 10$ principle (e.g., if 65 + 19 = 84, 75 + 19 must be 84 + 10 or 94), and the $N - 1$ principle for addition (e.g., if 52 + 27 = 79, 52 + 26 must be 79 − 1 or 78). Two inverse principles were investigated: for subtraction, the Addition/Subtraction Inverse principle (e.g., if 73 + 15 = 88, 88 − 15 must be 73) and for division, the Multiplication/Division Inverse principle (e.g., if 3 × 13 = 39, 39 ÷ 3 must be 13). For multiplication, the $M + 1$ principle was investigated (e.g., if 9 × 18 = 162, 9 × 19 must be 162 + 9 or 171). MacCuish (1986) found that 9- and 10-year olds would often answer one number above the answer in the initial multiplication fact, showing an overextension of the $N + 1$ addition principle to multiplication. Campbell and Graham (1985) found that errors by children and adults would often be out by a multiple of one of the operands. This principle was included to investigate whether RR would repeatedly make either type of error.

Participants were all tested individually in a quiet room. Prior to carrying out the experimental task, participants were given a pretest in each of the four operations to familiarize themselves with the tasks and to ensure a sufficient level of performance before starting the experiment. Each pretest consisted of 20 questions graduated in difficulty, including each of the different types of questions comprising the derived fact strategy tests (two- and three-digit addition and subtraction, single- and multi-digit multiplication and division, carrying and no-carrying addition questions, borrowing and no-borrowing subtraction questions).

In the derived fact strategy tests, each arithmetical question was presented with a corresponding fact directly above it. The fact (e.g., "21 + 53 = 74") allowed participants to quickly answer the question presented below it (e.g., "31 + 53 = ?") by applying the particular principle that was being investigated. Arithmetical problems that were numerically unrelated to the fact were used as control questions. There were equal numbers of control and experimental condition trials. Within each operation, the set of facts was kept the same across the different principles and control questions, so that half of the questions could be solved using the principle under consideration, and each question that involved a principle had a corresponding control question that used the same fact. Participants were instructed to press the spacebar when they knew the answer to the question, causing the question and fact to be replaced by a blank screen where they could enter their answer. This method of input allowed response times to be recorded on the decision time, and did not include any confounds of having to find the corresponding keys to the response (for a similar method and for a validation of the procedure, see, e.g., Atkinson, Campbell, & Francis, 1976; Demeyere, Rotshtein, & Humphreys, 2012; Watson & Humphreys, 1999). Participants were instructed to always look at the fact before answering the question, and to only press the spacebar when they knew the answer. They were asked to answer as quickly and as accurately as possible. Each of the six derived fact strategy tests consisted of 48 questions (288 in total) presented in four blocks of 12 questions, with a break between each block. The break lasted until the participants chose to carry on by pressing the spacebar.

Participants were given a practice derived fact strategy test of 16 questions involving two simple addition principles: the Commutativity principle (e.g., if 41 + 33 = 74, 33 + 41 must also be 74) and the $N + 1$ principle (e.g., if 14 + 68 = 82, 14 + 69 must be 82 + 1 or 83). After this practice task, they were given six derived fact strategy tests (each including 48 questions) investigating the principles given in Table 2.

In each of the six tests, two principles were investigated and the questions were presented in a random order. The first and third tests contained only addition questions, investigating the use of the $N + 3$ and $N + 10$ principles. The second and fourth tests contained both addition and subtraction questions, investigating the use of the $N - 1$ and Addition/Subtraction Inverse principles. The fifth and sixth tests contained both multiplication and division questions, investigating the $M + 1$ and Multiplication/Division Inverse principles. The total tasks took approximately 90 minutes to complete.

Results

Ten questions in the carrying/borrowing condition (seven addition, three subtraction) were omitted from the analysis as they involved two carries or borrows instead of one.

Healthy controls

Healthy controls showed effects of all principles, but to varying extents. A 2 × 6 Repeated Measures ANOVA with factors Strategy Use (Strategy/No Strategy) and Principles ($N + 3$, $N + 10$, $N - 1$, Addition/Subtraction Inverse, Multiplication/Division Inverse, and $M + 1$ principles) revealed a main effect of Strategy Use ($F(1, 15) = 55.53$, $p < .001$, partial $\eta^2 = .79$). There was a main effect of Principle ($F(5, 75) = 7.30$, $p = .003$, partial $\eta^2 = .33$) and a significant interaction between Strategy Use and Principle ($F(5, 75) = 5.51$, $p = .004$, partial $\eta^2 = .27$).

Paired samples *t*-tests were carried out comparing controls' accuracy and response times with and without the use of a principle. The results of these tests are presented in Table 3.

Table 3. Significant differences in response time (RT) and accuracy with and without a principle in the control group.

Principle	Mean score with principle (SD)	Mean score without principle (SD)	$t(15)$	p
N + 3 RT	4570 ms (2312 ms)	8130 ms (3125 ms)	7.93	<.001
N + 3 accuracy	96.20% (3.13%)	91.58% (6.82%)	−2.57	.021
N + 10 RT	4324 ms (1812 ms)	7782 ms (3345 ms)	7.04	<.001
N + 10 accuracy	97.16% (3.27%)	91.67% (8.61%)	−2.29	.037
N − 1 RT	4411 ms (1640 ms)	8332 ms (3326 ms)	6.95	<.001
N − 1 accuracy	92.97% (6.03%)	91.93% (6.17%)	−0.46	.652
ASI RT	4765 ms (1754 ms)	9887 ms (4267 ms)	5.84	<.001
ASI accuracy	96.74% (2.51%)	86.36% (8.78%)	−4.95	<.001
MDI RT	3373 ms (2576 ms)	6212 ms (3542 ms)	4.69	<.001
MDI accuracy	97.67% (2.62%)	91.93% (6.71%)	−3.02	.009
M + 1 RT	5327 ms (3483 ms)	7193 ms (4647 ms)	2.72	.016
M + 1 accuracy	93.49% (5.48%)	89.06% (8.98%)	−1.79	.094

Controls performed significantly faster and more accurately when using strategies based on the *N* + 3, *N* + 10, Addition/Subtraction Inverse, and Multiplication/Division Inverse principles than without such strategies. The *M* + 1 and *N* − 1 principles significantly reduced response times, but did not have a significant effect on accuracy.

RR

RR showed effects of five of the six principles (*N* + 3, *N* + 10, *N* – 1, Multiplication/Division Inverse, and *M* + 1), but to varying extents. Performance was analyzed using Crawford et al.'s (Crawford, Garthwaite, & Porter, 2010) test for dissociations in single-case studies, which uses inferential statistical methods to identify possible deficits and dissociations. Crawford and Howell's (1998) modified *t*-test (one-tailed) identifies whether a patient shows a deficit on a task (if they have a lower score compared to healthy controls). Response times were therefore converted to negative values in order to run this statistical test. The test for a dissociation also incorporates Crawford and Garthwaite's (2005) Revised Standardised Difference Test (RSDT), which determined whether the difference between RR's standardized *z*-scores was statistically significant. The computer program Dissocs_ES.exe (Crawford et al., 2010) was used to conduct these analyses.

RR's response times compared to controls' response times with and without the opportunity to use derived fact strategies are presented in Figure 3. RR exhibited a dissociation between response times for problems with and without the opportunity for using the *N* + 3, *N* + 10, and Multiplication/Division Inverse principles. Detailed descriptions of all comparisons run are given in Table 4. As advocated by Crawford et al. (2010), results (including effect sizes) are shown for all six principles despite the fact that only three (*N* + 3, *N* + 10, and Multiplication/Division Inverse) showed a statistically significant dissociation. Table 5 provides the point estimate and

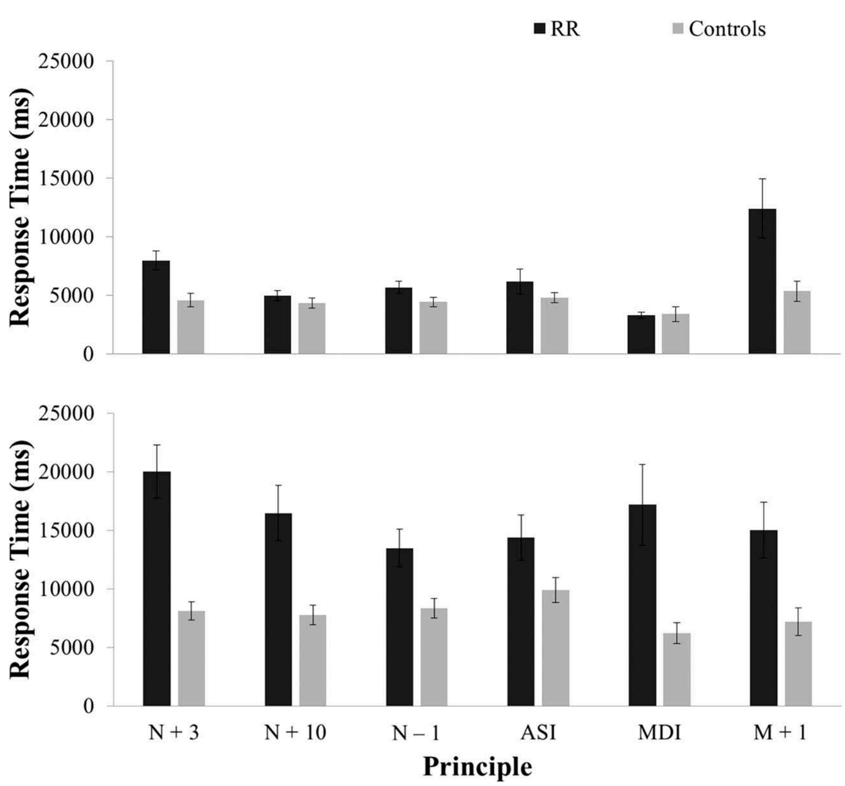

Figure 3. Response times (with standard error bars) of RR and controls with (top graph) and without (bottom graph) the use of derived fact strategies.

Table 4. Response times (RT) of controls and RR with and without use of a principle, the one-tailed significance level of Crawford and Howell's (1998) test for a deficit on the task without a principle, and the two-tailed significance level of Crawford and Garthwaite's (2005) Revised Standardised Difference Test.

Principle	Control sample RT (ms) without principle		Control sample RT (ms) with principle		RR's RT (ms) without principle	RR's RT (ms) with principle	Significance test of deficit without principle		Significance test on RSDT	
	Mean	SD	Mean	SD			$t(15)$	p	$t(15)$	p
$N + 3$	8130	3125	4570	2313	20020	7965	−3.69	.00109	3.51	.00317
$N + 10$	7782	3345	4324	1812	16460	4957	−2.52	.01184	3.98	.00121
$N - 1$	8832	3326	4411	1641	13467	5663	−1.50	.07746	1.11	.28556
ASI	9887	4267	4765	1754	14383	6174	−1.02	.16147	0.26	.80114
MDI	6212	3542	3373	2576	17183	3296	−3.01	.00444	3.81	.00171
$M + 1$	7193	4647	5327	3483	14999	12392	−1.63	.06201	0.51	.61442

Table 5. Effect sizes for the difference in response time (RT) between RR and controls, and estimates of abnormality.

Principle	Estimate of abnormality of RR's RT[a, c]		Estimated effect size (z_{cc})[d]		Estimate of abnormality of RR's discrepancy[b, d]	Estimated effect size (z_{cc})[d]	
	Point	*(95% CI)*	*Point*	*(95% CI)*		*Point*	*(95% CI)*
$N + 3$	0.11	(0.00 to 0.90)	−3.804	(−5.230 to −2.364)	0.16	−3.927	(−5.987 to −2.105)
$N + 10$	1.18	(0.01 to 6.13)	−2.595	(−3.626 to −1.544)	0.06	−4.491	(−6.437 to −2.815)
$N - 1$	7.75	(1.17 to 21.25)	−1.544	(−2.267 to −0.798)	14.28	−1.216	(−2.103 to −0.399)
ASI	16.15	(4.86 to 33.49)	−1.054	(−1.659 to −0.426)	40.06	−0.279	(−0.938 to 0.362)
MDI	0.44	(0.00 to 2.96)	−3.097	(−4.290 to −1.887)	0.09	−4.255	(−6.127 to −2.649)
$M + 1$	6.20	(0.74 to 18.48)	−1.680	(−2.439 to −0.897)	30.72	0.565	(−0.453 to 1.612)

[a]Estimated percentage of the control population obtaining a higher response time than RR without principle. [b]Estimated percentage of control population exhibiting a discrepancy more extreme than RR. [c]Crawford and Garthwaite (2002). [d]Crawford et al. (2010).

95% confidence interval (CI) of the abnormality of RR's response time (the estimated percentage of the control population that would obtain a higher response time than RR). It also shows the effect size (z_{cc}) for the difference between RR and controls (and its 95% CI), and the point estimate, 95% CI and effect size (z_{cc}) of the abnormality of RR's discrepancy between the tasks with and without a principle (the estimated percentage of the control population that would exhibit a discrepancy more extreme than RR).

For single-digit multiplication, RR showed higher response times than controls, both with and without the $M + 1$ principle. This was a differential deficit as he was significantly faster on trials involving the principle, fulfilling Crawford, Garthwaite, and Gray's (2003) criteria for a strong dissociation (showing a deficit compared to controls on both tasks, and being significantly worse at one of the tasks compared to the other). RR showed a deficit in response time without the $M + 1$ principle (M = 8721 ms, SD = 5759 ms) compared to controls (M = 3332 ms, SD = 2696 ms), $t(15) = -1.94$, $p = .03575$ (one-tailed), $z_{cc} = -1.999$ (95% CI = −2.849 to −1.127). RR also showed a deficit in response time with the $M + 1$ principle (M = 11781 ms, SD = 10274 ms) compared to controls (M = 2994 ms, SD = 1733 ms), $t(15) = -4.92$, $p = .00009$ (one-tailed), $z_{cc} = -5.071$ (95% CI = −6.925 to −3.204). RR's standardized response time without availability of the $M + 1$ principle was significantly higher than his response time with the principle, $t(15) = 5.00$, $p = .00016$, $z_{cc} = 5.724$ (95% CI = 3.202 to 8.606).

RR was not significantly faster on trials involving the $N - 1$ principle, but he made significantly more errors when the $N - 1$ principle was unavailable (M = 79.17%) compared to controls' accuracy with no available principle (M = 91.93%, SD = 6.17%), $t(15) = -2.01$, $p = .03168$ (one-tailed), $z_{cc} = -2.067$ (95% CI = −2.937 to −1.175). RR did not make significantly more errors on trials involving the $N - 1$ principle (M = 91.67%) compared to controls on this task (M = 92.97%, SD = 6.03%), $t(15) = -0.21$, $p = .41848$ (one-tailed).

RR also showed a dissociation between response times on addition questions involving carrying and the use of principles (including the three addition principles, $N + 3$, $N + 10$, and $N - 1$) and addition questions involving carrying without principles. RR was significantly slower without principles (M = 18081 ms, SD = 9102 ms) compared to controls (M = 9937 ms, SD = 4313 ms), $t(15) = -1.83$, $p = .04344$ (one-tailed), $z_{cc} = -1.888$ (95% CI = −2.707 to −1.048). RR was not significantly slower with principles (M = 5676 ms, SD = 2787 ms) compared to controls (M = 4480 ms, SD = 1958 ms), $t(15) = -0.59$, $p = .28110$ (one-tailed). There was a significant difference between RR's standardized response times on the two tasks, $t(15) = 2.44$, $p = .02757$, $z_{cc} = -2.711$ (95% CI = −4.027 to −1.549).

RR did not differ significantly in accuracy on addition, subtraction, or division questions from controls. He made significantly more errors in multiplication (M = 77.08%) compared to controls (M = 91.28%, SD = 5.55%), $t(15) = -2.48$, $p = .02544$, $z_{cc} = -2.557$ (95% CI = −3.578 to −1.518).

Discussion

The results support the first hypothesis that both RR and healthy controls would answer mental arithmetic questions faster when there was an opportunity to use a derived fact strategy than when there was no such opportunity. Controls performed significantly faster using strategies based on each of the six principles compared to trials where the arithmetical fact presented was numerically unrelated to the question. RR showed such effects for five of the six principles. He was faster when given the opportunity to use the $N + 3$, $N + 10$, and Multiplication/Division Inverse principles, the $M + 1$ principle in single-digit multiplication, and the $N + 3$, $N + 10$ and $N - 1$ principles for addition questions that required carrying. The only principle whose availability did not lead to faster performance by RR was the Addition/Subtraction Inverse principle for subtraction. Although RR was slower than controls in addition, division and multiplication when there was no opportunity to use a derived fact strategy, he performed most complex, multi-digit mental arithmetic tasks as accurately as controls, despite his apparent lack of a phonological loop and his severe expressive aphasia which rendered him incapable of verbalizing responses or counting aloud past the number 13.

The controls showed highly significant differences in response times for problems that were and were not preceded by a fact that allowed use of a principle, especially for addition, subtraction, and division. Controls also performed more accurately when using the $N + 3$, $N + 10$, Addition/Subtraction Inverse, and Multiplication/Division Inverse principles. The $M + 1$ principle did not have a significant effect on controls' accuracy, perhaps because multiplication problems tend to be solved using fact retrieval (Hittmair-Delazer et al., 1994; Lee & Kang, 2002).

The second hypothesis that RR would show a significantly larger difference in response times with versus without the use of derived fact strategies compared to healthy controls, was supported for five of the six principles. Crawford et al. (2003) provided an operational definition of a classical dissociation as the patient performing significantly worse than controls on Task X, the patient not performing significantly worse than controls on Task Y, and the patient's performance on Task X being significantly worse than their performance on Task Y. For the $N + 3$, $N + 10$, and Multiplication/Division Inverse principles, RR performed significantly worse than controls when there was no opportunity to use the principles (meeting the criterion for a deficit). In contrast, RR did not perform significantly worse than controls when there was an opportunity to use these principles, and his performance without the principles was significantly worse than his performance with the principles. While this pattern of performance meets Crawford et al.'s (2003) definition of a classical dissociation, Crawford and Garthwaite (2006) instead describe this type of dissociation as 'putatively classical' to indicate that although their test can confidently show that a patient has suffered some type of dissociation, patients categorized as exhibiting classical dissociations may actually be cases of strong dissociation (i.e., there was also a deficit on the task with principles that was not detected, but more of a deficit without principles, constituting a differential deficit).

Given that RR was within normal limits when using the $N + 3$, $N + 10$, and $N - 1$ principles for addition questions that involved carrying, he demonstrated a putatively classical dissociation in response times between carrying questions with the availability of derived fact strategies and those without. Previous research has suggested that the central executive is the main working memory component involved in carrying (Fürst & Hitch, 2000; Imbo et al., 2007) which could account for the fact that RR did not show a significant difference in accuracy compared to controls for carrying questions. However, both of these studies did show some role of the phonological loop in carrying, which the findings of this study support, as RR was significantly slower when carrying without the availability of a strategy.

For single-digit multiplication, RR exhibited a strong dissociation in response times with and without availability of the $M + 1$ principle. This is consistent with previous findings that single-digit multiplication tends to rely on fact retrieval, but patients with impaired fact retrieval, associated with aphasia, can use back-up strategies to allow them to perform such calculations (Hittmair-Delazer et al., 1994; McCloskey et al., 1985). RR's use of the $M + 1$ principle indicates he has some conceptual knowledge of this operation and principle (he did not make the types of errors reported by MacCuish, 1986; or Campbell & Graham, 1985). RR was found to be within normal limits of response times and accuracy for all types of subtraction questions. This supports Lee and Kang's (2002) findings, which suggested that subtraction relies more on the visuo-spatial sketchpad than the phonological loop. Seyler et al. (2003) argued for the involvement of the phonological loop in larger, multi-digit subtraction questions. However, the results of the present study suggest that the role of the phonological loop may be relatively small for subtraction.

These results indicate that when RR used derived fact strategies based on the $N + 3$, $N + 10$, and Multiplication/Division Inverse principles, and the $N - 1$ and $M + 1$ principles in certain conditions, he was able to compensate for his severe working memory and language deficits to the extent that he performed within normal limits of response times. Without availability of these principles, RR was significantly slower than controls. He was also less accurate than controls for multiplication but did not differ significantly from controls in accuracy for addition, subtraction, or division problems (with or without availability of a derived fact strategy). This finding calls into question the necessity of the phonological loop in multi-digit arithmetic (e.g., Noël et al., 2001) and in allowing individuals to maintain accuracy in mental arithmetic (e.g., Logie et al., 1994). It supports the results of Butterworth, Cipolotti, and Warrington (1996), who also reported a patient with an impaired phonological loop but preserved arithmetical skills, though they did not specifically examine the role of derived fact strategies.

This is not to say that the phonological loop does not play an important role in mental arithmetic; there is strong evidence to suggest that it is necessary for maintaining operands when they are presented only briefly (Fürst & Hitch, 2000; Heathcote, 1994; Noël et al., 2001), which was not the case in this experiment. Nevertheless, the performance of RR on complex multi-digit questions in addition, subtraction, and division suggests that the phonological loop is not essential for multi-digit calculations in these operations.

The findings support the idea that there are alternative ways of maintaining interim results in working memory, besides the phonological loop (e.g., Heathcote, 1994; Noël et al., 2001), supporting the findings of Logie et al. (1994). RR appeared to make deliberate use of non-phonological methods of maintaining information. On one occasion, he reported using the spatial arrangement of the numeric keypad (which displays the numbers 1–9 in a 3 × 3 arrangement, with the number 0 on a separate row at the bottom) to represent numbers using directions. He would keep his middle finger on the number 5 (in the center of the keypad) and represent other numbers in relation to this (e.g., "right" for 6). RR stated that he did not need the physical keypad in front of him to use this strategy, as he could visualize the numeric keypad in his head and apply this strategy in different situations. This method allowed RR to represent paths on the keypad as shapes (e.g., he thought of 297 as a triangle). This representation provides support for Logie's (1995, 2011) description of an "inner scribe" in visuo-spatial working memory, allowing individuals to maintain short sequences of movements through mental rehearsal.

In addition to this visuo-spatial representation of numbers, RR also reported linking certain numbers on the numeric keypad using symmetry. For example, if he had to calculate 10 – 3, RR had established that the answer was the number diagonally opposite 3 (7). Other pairs of numbers were also linked in this way (e.g., 6 is opposite 4).

Although RR had suffered large left temporal lesions extending into his frontal and parietal cortex, resulting in severe expressive aphasia, he was able to apply numerous derived fact strategies in addition, division, and to a lesser extent multiplication, which suggests that such strategies do not rely on these lesioned areas per se. In particular, the left angular gyrus, which was completely lesioned here, has long been implicated in studies on calculation and the retrieval of arithmetic facts (e.g., Grabner et al., 2009). In line with this, RR indeed also demonstrated impaired multiplication performance (due to impaired verbal fact retrieval), though we note that he was distinctly not at floor and still able to do most multiplications. Damage to the angular gyrus has been linked to Gerstmann's syndrome, where acquired dyscalculia co-occurs with finger agnosia, left–right disorientation, and agraphia (Gerstmann, 1940). It is clear from these data that there is no unambiguous link between the behavioral disorder of dyscalculia and left angular gyrus damage. RR did present with associated dysgraphia, but not dyscalculia.

The findings of this study suggest that RR was able to use various derived fact strategies to overcome his severe working memory and language deficits, and accurately carry out mental arithmetic at a speed within normal limits of healthy, age-matched controls. Thus, the study supports and extends earlier research (Hittmair-Delazer et al., 1995; Warrington, 1982) in demonstrating a striking dissociation between factual and conceptual knowledge: specifically fact retrieval and derived fact strategies. This is important evidence for the view that arithmetical cognition is componential (Delazer, 2003; Dowker, 1998); that the different components are potentially functionally independent; and that significant impairment in one component, in this case fact retrieval, may be compatible with generally good arithmetical performance, if other components are used to compensate.

These findings could have implications for stroke rehabilitation programs. It is important to investigate whether other stroke survivors with verbal working memory and fact retrieval deficits could also benefit from applying derived fact strategies in arithmetic.

Researchers could also investigate whether teaching derived fact strategies to children with retrieval difficulties arising from verbal working memory deficits would help them compensate for their deficits in a similar way to RR. There is anecdotal evidence that children with dyslexia, who often have associated phonological memory deficits and specific problems with arithmetical fact retrieval, sometimes use derived fact strategies to compensate (Miles, 1993; Steeves, 1983). It would be desirable to investigate the possibilities for harnessing this ability through systematic training in children with verbal working memory difficulties, especially as there is already evidence that training in the use of derived fact strategies can improve performance in children with mathematical difficulties (Askew, Bibby, & Brown, 2001).

This study showed that despite suffering large left temporal lesions, RR was able to implement a variety of derived fact strategies in addition, division, and multiplication, which suggests that certain strategies do not rely on left temporal regions. This could, to some extent, help further research into the neurological underpinnings of derived fact strategies. Future studies could incorporate brain imaging techniques in healthy adults to investigate the different areas involved in derived fact strategy use.

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