

## OBSERVATION

# Immediate Small Number Perception: Evidence From a New Numerical Carry-Over Procedure

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Evidence is presented for the immediate apprehension of exact small quantities. Participants performed a quantification task (are the number of items greater or smaller than?), and carry-over effects were examined between numbers requiring the same response. Carry-over effects between small numbers were strongly affected by repeats of pattern and number identity relative to when displays were from the same response category but contained different numbers. Carry-over effects with large items were less sensitive to both pattern and number identity, even when the numbers in the small and large categories were matched for discriminability. The data suggest that small numbers are immediately apprehended through a direct subitization process distinct from pattern recognition and the apprehension of approximate number.

*Keywords:* numerical cognition, subitizing, estimation

When perceiving nonsymbolic numerosities, one can either count the elements to determine the exact value or estimate the quantity for an approximate representation of the magnitude. The exact enumeration of small numbers is fast with only a small increase in enumeration time across items (50–80 ms), whereas for the larger numerosities there is a linear response time (RT) increase for every enumerated item (about 200 ms/item) (Mandler & Shebo, 1982; Trick & Pylyshyn, 1993, 1994). This difference in performance is the basis for the distinction between “subitizing” (the ability to enumerate in a fast and accurate manner a small groups of objects) and “counting” (the process of serially counting larger numbers of items). In estimation, larger numbers can only be estimated approximately, whereas small numbers are estimated exactly.

It is still controversial as to whether subitizing is a special process, distinct from the operations involved in counting and estimation. For example, Feigenson, Dehaene, and Spelke (2004) propose separate processes representing approximate and exact number, with this second system in subitization. In contrast, others

have argued that subitization and estimation use the same processes which operate with higher resolution or more rapidly on smaller numbers (Dehaene & Changeux, 1993; Ross, 2003; though see Revkin, Piazza, Izard, Cohen, & Dehaene, 2008). The argument for distinct processes has not been resolved by data from brain imaging, with some reports documenting overlapping neural areas for counting and subitizing (Sathian et al., 1999; Piazza, Giacomini, Le Bihan, & Dehaene, 2002; Piazza, Mechelli, Butterworth, & Price, 2003), while others have found distinct regions recruited when subitization takes place (Ansari, Lyons, van Eimeren, & Xu, 2007; Vetter, Butterworth, & Bahrami, 2011).

In this study, we present a new procedure, based on carry-over effects across trials in a quantification task, to compare the subitization of small and the estimation of larger numbers. Importantly, we control for the discriminability of the numerosity displays, by having a similar Weber ratio between the numerosities in both categories (small and larger numbers). There is a long history of research showing that responses are faster both when properties of the stimulus repeat and when the same response is required (Bertelson, 1961; Pashler & Baylis, 1991; Rabbitt, 1968; Smith, 1968). The contrast between different stimulus repetition conditions, in particular, can be informative about the nature of the stimulus representations and the stimulus-response mappings mediating performance. We capitalized upon this to examine number processing when consecutive displays required the same response (either both small or both large). Participants were asked to decide whether a presented quantity was greater or less than a set number. With both the smaller and larger numerosities, consecutive trials could have (i) repeats of an identical pattern (the pattern repetition condition), (ii) repeats of the same number identity, but in a different pattern (number repetition trials), and (iii) repeats of a different quantity but from the same response category (category repetition trials). Given that these conditions all involve the same response, any contrast in the size of the repetition effects cannot be attributable

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to differences in response selection but must reflect the processes involved in numerosity judgments. We assessed whether carry-over effects varied for small and large numbers. Carry-over effects have been used previously to explore numerosity judgments in both behavioral studies (Koechlin, Naccache, Block & Dehaene, 1999) and neuroimaging (e.g., Ansari, Dhital & Siong, 2006; Piazza, Mechelli, Price, & Butterworth, 2006), but this work has typically examined effects of numerical distance and notation (symbolic vs. nonsymbolic numbers), not the contrast between small and large numbers.

We reasoned as follows. If different processes are engaged across consecutive trials, then carry-over effects will reduce relative to when the same processes are engaged. Notably, the rapid apprehension of different number values within the subitization range may lead to reduced carry-over effects for smaller number values requiring the same response when compared with larger number values, where common (approximate) number values are computed for different sizes of display. In contrast, there may be larger carry-over effects in the pattern and number repetition conditions, through recognition and/or subitization of a common number identity. It has been argued that there is rapid apprehension of small number through recognition of their patterns (Logan & Zbrodoff, 2003; Mandler & Shebo, 1982). Are there differential carry-over effects based on the similarity of consecutive patterns, for small relative to large numbers, when displays are controlled for discriminability?

## Method

We used Weber-matched differences between the stimuli within the small and large numerosities. In the small category we used displays with 2, 3, or 4 dots; in the large category the displays contained 6, 8, or 11 dots, which have Weber discriminability ratios between 0.75 and 0.80 (see Shuman & Kanwisher, 2004; Nieder & Dehaene, 2009;). The dots were made by combining binary noise with a circular Gaussian envelope. The diameter of each dot was 30 pixels, on a screen with  $1280 \times 1024$  pixel resolution. The dots were drawn randomly in the center of the display, within a  $450 \times 450$  pixel window, with the constraint of

a minimum interdot distance and distance from the fixation cross of 30 pixels. To avoid systematic variation of luminosity with numerosity, the dots were randomly sampled (with replacement) from a list of 10 elements. These 10 elements were measured by a Minolta LS110 light meter, to fall within a range of  $1 \text{ cd/m}^2$  from the average background luminosity. The luminosity of the background measured  $12.2 \text{ cd/m}^2$ , the dots in the list were chosen so that there were four items “darker” than the background with luminosity values:  $11.2 \text{ cd/m}^2$ ,  $11.7$ ; two similar to the background:  $\text{cd/m}^2$ ,  $12.2 \text{ cd/m}^2$ , and four with brighter luminosity than the background:  $12.7 \text{ cd/m}^2$ ,  $13.2 \text{ cd/m}^2$ . These values were measured on a high-resolution CRT monitor in a completely darkened room. By sampling the elements in this way, luminosity was not the same in each display across all numerosities, but there was no consistent relationship in which larger displays always have a larger luminosity than smaller displays (e.g., it is possible that a display numerosity 8 could be “lighter” or “darker” than a display with numerosity 2), and on average the luminosity was the same across the number conditions. The dots appeared on a gray background (RGB: 127,127,127). For an example of the stimuli used, see Figure 1.

There were two response categories: small and large. Participants responded “small” (key 1 on the numeric pad) when there were fewer than five dots and “large” (key 2 on the numeric pad) when there were more than five dots present. There were three types of repetition: (i) same pattern (consecutive stimuli were exactly the same in both number and position); (ii) same number identity (consecutive displays had the same number of elements but their locations were randomized across trials); and (iii) same category (the items present differed in number but remained in the same response category). In addition, in a fourth pairing consecutive displays had different numbers of dots drawn from different response categories. Repetition bias was avoided by having more trials where no response repetition was required compared with repetition trials. Second order repetitions (e.g., runs of three or more items from the same category) were reduced by using a pseudorandom order list with a fixed number of trials per condition. There were 16 trials for each numerosity for identical, number, and category repetitions, and 107 trials for each numerosity

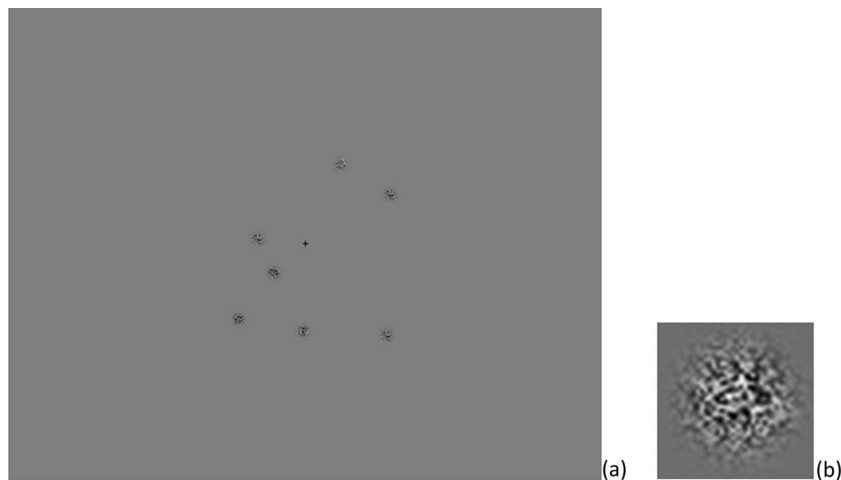


Figure 1. (a) Example of a stimulus in the “large” category. (b) Close-up of a single dot.

where there was no repetition of the response category. Sixteen participants from the University of Birmingham took part voluntarily for research credits. They received a total of 786 trials divided over six blocks with breaks after each block.

## Results

The correct RTs are presented in Figure 2. The data were analyzed in a three-factor repeated measures ANOVA with the factors being repetition type, numerosity, and response category. Most critically, there was a significant interaction between the repetition types and the response categories (small vs. large numbers –  $F[2, 30] = 9.686, p = .002$ , partial  $\eta^2 = .392$ ) and a reliable three-way interaction ( $F[4, 60] = 2.971, p = .042$ , partial  $\eta^2 = .165$ ). Note that there was no reliable main effect of the response category itself (small vs. large –  $F[1, 15] = 2.986, ns$ ).

The contrast between the same pattern and category conditions interacted with the response category (small vs. large displays –

$F[1, 15] = 19.773, p < .001$ , partial  $\eta^2 = .569$ ). There was also an interaction between the response category and the contrast between the same number and category conditions,  $F(1, 15) = 6.248, p = .025$ , partial  $\eta^2 = .294$ , demonstrating that, relative to the same category condition, there were stronger pattern and same number carry-over effects for small over large displays. For both small and large displays, RTs were faster to same pattern than to same number displays,  $F(1, 15) = 60.294, p < .001$ , partial  $\eta^2 = .801$  and  $F(1, 15) = 46.529, p < .001$ , partial  $\eta^2 = .756$ , and RTs in these number repetition trials were in turn faster than RTs to same category displays,  $F(1, 15) = 24.720, p < .001$ , partial  $\eta^2 = .622$  and  $F(1, 15) = 9.314, p = .008$ , partial  $\eta^2 = .383$ . Though both response categories (small and large numbers) showed carry-over effects, the critical result is that these effects were much larger for small, subitizable numbers than for larger numbers. Given that display magnitudes within the small and large categories had similar Weber fractions, the contrasting effects cannot be attrib-

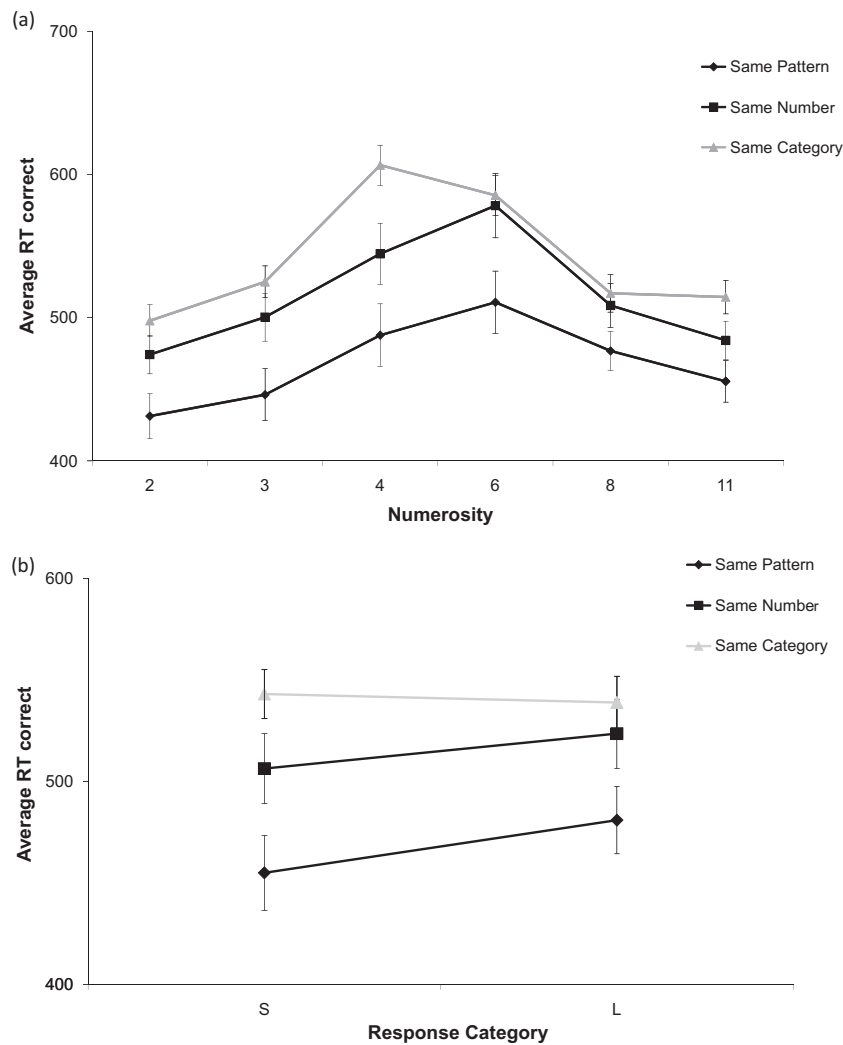


Figure 2. RTs (ms) on correct responses. (a) Average RTs with standard error bars for each numerosity and repetition type for the small and large response categories. (b) Mean RTs and standard error bars for each repetition type and each response category.

uted to the lack of discriminability within the large number set. Instead the data fit with the idea that small numbers are computed exactly, so that same pattern and same number identity repeats are perceived as being more similar than same category repeats. The results contrast with this for displays with larger numbers. The more similar performance for the same number identity and category conditions with larger numbers indicates that approximate rather than exact representations of individual items were computed. When the pattern changed, there was little difference according to whether the exact number of items was preserved.

### Discussion

There were differential carry-over effects for small and large numbers in a visual enumeration task. For small numbers, clear differences emerged between the different carry-over conditions: RTs on same pattern trials were faster than on same number trials, which were in turn faster than on category repeat trials. For larger numbers the differences between the number identity and same category trials were minimal, although effects of pattern carry-over occurred. Demeyere (2010) replicated these findings with a wide range of displays and using stimuli which were matched for the similarity of consecutive patterns in the small and large number categories, making it unlikely that these results can be attributed to the small patterns appearing more dissimilar from one another than the larger patterns (and so generating weaker carry-over effects in the same category condition). We conclude that the differential effects in the number and category conditions, for small and large numbers, reflect the carry-over of common subitization processes on consecutive trials with small number displays. Because larger number displays cannot be subitized and are only processed approximately, the differential carry-over effects are absent for these displays. One argument is that subitization is based on the parallel allocation of a limited number of FINSTs ('Fingers of Instantiation'; Trick & Pylyshyn, 1993), which mark the locations of perceptual objects. The limited number of FINSTs prevents their application to all of the items making up displays with larger numbers. Whether this is the case or not, our results also suggest that subitization processes (absent with large number displays) are distinct from pattern recognition, given that pattern effects emerged for large number displays. This conclusion is supported by neuropsychological evidence indicating that, for example, patients can be extremely poor at counting even small numbers (because of impaired subitization), even when pattern recognition processes remain relatively preserved (e.g., in cases of severe simultanagnosia; see Humphreys, 1998).

Although they suggest that subitization processes exist for small displays, our results still implicate effects of pattern recognition in enumeration. We do not think that effects of pattern recognition and subitization are mutually exclusive; rather, both may contribute to visual enumeration. Our data, however, provide a basis for distinguishing between pattern recognition and subitization effects, as well as demonstrating conditions in which enumeration of exact small numbers differs from approximate coding of larger numbers. In this respect it is again interesting to note the neuropsychological evidence indicating that patients with impaired subitization can nevertheless compute large approximate numbers (Demeyere & Humphreys, 2007; Demeyere, Rzeskiewicz, Humphreys, & Humphreys, 2008). In this respect our results converge with the neu-

ropsychological evidence in indicating that pattern recognition, enumeration, and approximate number processing exist as separate processes.

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